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Measuring Competitiveness in Twos

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Abstract

Improvements in competitiveness can be achieved through policy initiatives, but the success of these policies will depend upon the way that firms and consumers respond. This paper establishes the conditions under which a policy change can lead to an improvement in the competitiveness of a Canadian firm. There are two firms (Canadian, U.S.) each with two brands and each making sales in two markets (Canada, U.S.) and two consumers, one in Canada and one in the U.S. Equilibrium is shown to depend on inverse compensated demand function coefficients, the conjectured best response coefficients for each firm and marginal cost functions for each firm. An improvement in competitiveness from an investment in public infrastructure in Canada is shown to depend upon initial sales ratios and the by sign and size of the best response of the Canadian firm as conjectured by the U.S. firm. It is also shown how the policy may have unintended effects. The model can be used to derive a range of other results and these potential uses are outlined.

Résumé

La compétitivité peut être accentuée par le biais de politiques, mais le succès de ces politiques dépend des réactions des firmes et des consommateurs. Nous dérivons les conditions pour qu'un changement de politique puisse améliorer la compétitivité d'une firme canadienne. Nous supposons que deux firmes, une canadienne et une américaine, se concurrencent sur les marchés canadiens et américains en offrant des marques différentes de produits sur chaque marché. L'équilibre dépend des fonctions inverses de demandes compensées, des conjectures entretenues par les firmes et des coûts marginaux des firmes. L'amélioration de la compétitivité découlant d'un investissement public en infrastructure est influencée par le ratio initial des ventes de même que par le signe et la taille de la réaction de la firme canadienne telle qu'anticipée par la firme américaine. Il est aussi démontré que l'investissement public peut avoir des conséquences non-attendues. D'autres résultats peuvent être dérivés à partir du modèle qui se prêtes à diverses utilisations qui font l'objet d'une discussion.

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1 Introduction

Competitiveness has been the subject of much discussion, both in terms of what it is and how to change it. In spite of its prominence in discussions around the economy and economic growth, there is apparently is no generally accepted economic model of competitiveness. Economists have mostly limited their attention to measurement, relying mostly on trade-based or cost/efficiency/productivity indicators (Latruffe, 2010, pp. 5-6). One problem with these indicators is that they are not linked in any purposeful way to economic theory.

The apparent absence of a well-developed theory of competitiveness hinders the design of effective policies and programs. If the nature and causes of competitiveness are not well defined, the social returns from public investments meant to improve competitiveness cannot be measured. This means that the design of programs meant to improve competitiveness and decisions about relative rates of funding for these programs must be made without information on relative expected rates of return.

Competitiveness is often characterized as the ability of a firm to increase sales in export markets. It is also frequently characterized as the ability of a firm to increase sales in the domestic market. Both characterizations are usually expressed in terms of the firm increasing sales when facing competition from other firms in either market. It is a given that a firm which exemplifies competitiveness is one that also remains profitable and/or becomes more profitable over time. It is also a given that firms are rivalrous by virtue of the word ‘competition’.

In spite of being at the top of government agendas for several decades, competitiveness is still defined in a myriad of ways and measurement of it remains bafflingly imprecise. As Latruffe observes, competitiveness “is ....a broad concept and there is no agreement on how to define it nor how to measure it precisely. There is a profusion of definitions with studies often adopting their own definition and choosing a specific method” (Latruffe, 2010, p. 5). The characterizations referred to above point to a need for an analysis that can take all of these elements into account. In particular, such an analysis must be able to address the fact that a firm will often be making sales in both the domestic and foreign market and therefore that it needs to be competitive in at least one of these two markets if it is to stay in business. This means that it is necessary to be precise about the meaning of competitiveness, with the cost of adding another definition to the mix. Here, since the focus is on improvements in competitiveness, the definition is expressed in terms of change rather than levels:

> **Definition of an improvement in competitiveness**
> For a firm, an improvement in competitiveness will be reflected by consistent sales growth in any relevant market at a rate that is at least as high as the sales growth of other firms (the competition) in that market.

A relevant market is taken here to be one in which the firm is already making sales. The analysis that follows could readily be extended to accommodate entry into new markets.

Irrespective of the definition used, any analysis of competitiveness must take the consumer into account. This aspect has typically been neglected, with most of the focus being on the firm, the industry or the market, where the latter is usually referred to in terms of size, rather than in terms of the consumers that comprise it. This apparent neglect of the consumer is surprising, given that consumers are ultimately the ones who determine the degree to which a firm will be able to grow, either by overall growth in demand (of which it takes some share) or by substitution towards its product/brand in favour of products/brands produced by other firms.

The purpose of this paper is to develop a model that can incorporate all of these features and that can measure the impact of a range of policies designed to improve competitiveness of Canadian firms. In many respects, the model presented here has quite routine features and is similar to existing models. It is a quantity-setting duopoly model, but with the two firms producing two products (brands) and selling them in two markets (Canada and United States) to two consumers, one in each market. There are at least two other models in the literature that have similar features to the one developed here – those of Bulow, Geanakopolos and Klemperer (1983) and Fung(1991). The multimarket oligopoly model presented in Bulow et al. provides a much more complete treatment of firm interactions but is more limited in terms of detail, particularly with respect to the role of policy, where their policy analysis is limited to subsidies. Fung's model, while similar, focusses on the existence and stability of various collusive arrangements. Neither model is directly used to measure competitiveness, although either of them could easily be oriented in that direction.

While firms in this model can, by definition, influence prices through their quantity-setting strategies, this analysis is not concerned with market power effects that come about from oligopoly. There is a large literature dealing with imperfect competition both with homogeneous and differentiated products – for food processing see for example Sexton and Lavoie(2001) or Wann and Sexton(1992). This literature focusses primarily on markups and the exertion of market power rather than the level of output and changes in it. These studies are typically concerned with interactions in one market rather than in two or more markets. There is similarly a large literature on trade in differentiated products – for food processing in particular see Sarkar and Surry(2006). Here, the firm usually plays no role at all – the focus is primarily on the industry and the consumer, with different industries in various countries selling a commodity that is differentiated only by origin.

This model can generate a range of testable implications, following the approach used in Panzar and Rose(1987), Bulow, Geanakopolos and Klemperer(1983) and Fung(1991), where a range of formal propositions point to areas where quantification might lead to further insights The potential range of testable implications is only touched on here – this is one of the areas that will emerge from subsequent further research related to the various displacements from equilibrium generated by hypothetical government programs.

2 Assumptions and Definitions

There are quite a few preliminaries that must be covered off before it is possible to outline the model and derive results with it. The following lays out the assumptions on structure, behaviour and rules as well as the notation needed for implementation.

2.1 Notation

- There are two firms; one is the Canadian firm (‘C’) – the first subscript in the notation – where the firm can be thought of as a plant situated in Canada.[1] The second is the U.S. firm (‘U’) – the first subscript in the notation – where the firm can be thought of as a plant situated in the United States

- There are two markets Canadian and U.S. (superscripts ‘c’ and ‘u’ respectively in the notation)

- There are two brands of a commodity produced by each firm and that commodity is differentiated by origin where :

  - y_C^c and p_C^c are production/sales and market price respectively for the brand produced by the Canadian firm and sold in the Canadian market

  - y_C^u and p_C^u are production/sales and market price respectively for the brand produced by the Canadian firm and sold in the U.S. market

[1] Ownership of the plant (and permanence of it) is relevant to policy but not addressed here. It is fair to say, though, that there are many possible types of ownership, such as: (i) the plant is privately owned by an individual who resides in Canada; or (ii) the plant is owned by a public company, shareholders of which all reside outside of Canada. The intent and extent of policy may differ if it is directed at one and not the other type of plant ownership.

- y_U^u and p_U^u are production/sales and market price respectively for the brand produced by the U.S. firm and sold in the U.S. market
- y_U^c and p_U^c are production/sales and market price respectively brand produced by the U.S. firm and sold in the Canadian market

- There are two representative consumers (one in each market) with sub-ututility z_C for the Canadian consumer and z_U for the U.S. consumer.

Other notation is introduced as needed.

2.2 Assumptions on Structure, Behaviour and Rules

2.2.1 Consumers

Consumers in the Canadian market substitute between the Canadian and U.S. brands. They are indifferent between the Canadian brand sold in the Canadian market and the Canadian brand sold in the U.S. market (for example because they differ trivially, e.g. by packaging size or labelling). The same is true for U.S. consumers – they are indifferent between the two U.S.-produced brands. Together, this means that cross-border purchases by consumers are ruled out. The brands are sufficiently different, however, for the Canadian firm to not sell its Canadian-marketed brand in the U.S. market, i.e. the markets are segregated.

Demand is represented using inverse compensated demand functions, which hold subutility fixed. It is important that inverse demand functions (rather than conventional demand functions) be used, because the effect of changes in quantity on price are determined through the demand side of the model. The choice of compensated rather than uncompensated functions is somewhat arbitrary, although from a policy analysis standpoint, it may be desirable to look at policy alternatives that have a neutral effect on consumer welfare. Relative prices must adjust to match the quantity of the two brands available on the market. It is assumed that the preference ordering of consumers is known to both firms, for example from publicly available market surveys.

Canadian consumers have inverse compensated demand functions κ^c_C and κ^c_U for Canadian and U.S. brands respectively, which are specified as:[2]

- $p^c_C = \kappa^c_C(y^c_C, y^c_U, z_C)$
- $p^c_U = \kappa^c_U(y^c_C, y^c_U, z_C)$

Similarly, U.S. consumers have the inverse compensated demand functions κ^u_C and κ^u_U that are specified as

- $p^u_C = \kappa^u_C(y^u_C, y^u_U, z_U)$
- $p^u_U = \kappa^u_U(y^u_C, y^u_U, z_U)$

The inverse compensated demand functions have several properties – see [5, p. 666] – which are stated here because it is possible to use them when establishing some testable implications with the model:

- Homogeneity of degree zero in y^c_C, y^c_U, which requires that

$$(\partial\kappa^c_C(y^c_C, y^c_U, z_C)/\partial y^c_C)\, y^c_C + (\partial\kappa^c_C(y^c_C, y^c_U, z_C)/\partial y^c_U)\, y^c_U = 0$$

  and

$$(\partial\kappa^c_U(y^c_C, y^c_U, z_C)/\partial y^c_C)\, y^c_C + (\partial\kappa^c_U(y^c_C, y^c_U, z_C)/\partial y^c_U)\, y^c_U = 0$$

  for the inverse compensated demand functions facing the Canadian firm in the Canadian and U.S. markets respectively and

$$(\partial\kappa^u_C(y^u_C, y^u_U, z_C)/\partial y^u_C)\, y^u_C + (\partial\kappa^u_C(y^u_C, y^u_U, z_C)/\partial y^u_U)\, y^u_U = 0$$

  and

$$(\partial\kappa^u_U(y^u_C, y^u_U, z_C)/\partial y^u_C)\, y^u_C + (\partial\kappa^u_U(y^u_C, y^u_U, z_C)/\partial y^u_U)\, y^u_U = 0$$

  for the inverse compensated demand functions facing the U.S. firm in the Canadian and U.S. markets respectively.

[2] Derivation of the inverse compensated demand functions is not required for this analysis and so it is omitted – see Kim(1997) for the derivations underlying these functions.

- Symmetry

$$\partial \kappa_C^c(y_C^c, y_U^c, z_C)/\partial y_U^c = \partial \kappa_U^c(y_C^c, y_U^c, z_C)/\partial y_C^c$$

and

$$\partial \kappa_C^u(y_C^c, y_U^c, z_C)/\partial y_U^u = \partial \kappa_U^u(y_C^c, y_U^c, z_C)/\partial y_C^u$$

- Concavity, which requires negative semi-definiteness of the matrices

$$E_C = \begin{bmatrix} \partial \kappa_C^c(y_C^c, y_U^c, z_C)/\partial y_C^c & \partial \kappa_C^c(y_C^c, y_U^c, z_C)/\partial y_U^c \\ \\ \partial \kappa_U^c(y_C^c, y_U^c, z_C)/\partial y_C^c & \partial \kappa_U^c(y_C^c, y_U^c, z_C)/\partial y_U^c \end{bmatrix}$$

and

$$E_U = \begin{bmatrix} \partial \kappa_U^u(y_C^u, y_U^u, z_U)/\partial y_U^u & \partial \kappa_C^u(y_C^u, y_U^u, z_U)/\partial y_C^u \\ \\ \partial \kappa_C^u(y_C^u, y_U^u, z_U)/\partial y_U^u & \partial \kappa_C^u(y_C^u, y_U^u, z_U)/\partial y_C^u \end{bmatrix}$$

with necessary conditions

$$\partial \kappa_C^c(y_C^c, y_U^c, z_C)/\partial y_C^c < 0 \text{ and } \partial \kappa_U^u(y_C^u, y_U^u, z_U)/\partial y_U^u < 0$$

respectively. A second necessary condition is that the determinants of E_C and E_U be non-negative. For this to hold, it is necessary that

$$\partial \kappa_U^c(y_C^c, y_U^c, z_C)/\partial y_U^c < 0 \text{ and } \partial \kappa_C^u(y_C^u, y_U^u, z_U)/\partial y_C^u < 0$$

respectively, since due to symmetry, the product of the off-diagonals of E_C and E_U is always positive. Note that the latter condition, while necessary, is not sufficient to ensure concavity, since it is possible for the product of the diagonal elements to be less than the product of the off-diagonal elements in each matrix.

2.2.2 Firms

The Canadian firm and U.S. firms are assumed to behave as quantity-setting duopolists where each maximizes profit by taking into account both the consumer response as well as the response of the other firm. The problem is one of finding the optimal quantity of sales for each brand in each market. As in Bulow et al., the firm's decision about how much to sell in each of the two markets is linked only through costs of production (Bulow, Geanakopolos and Klemperer, 1983, p. 17). In this model, that link comes through fixed inputs and exogenous variables (some of which are set by policies) that are shared in the production of each brand. Once each firm has solved for the optimal level of sales it is possible to use the properties of the optimal solutions and related demand responses to measure competitiveness and look at how this changes when the exogenous policy variables change.

The assumptions regarding structure, behaviour and related 'rules of the game' are now outlined:

- the Canadian firm produces y_C^c and y_C^u using the same technology – where the two brands may involve different combinations of inputs (such as packaging) – which can be represented as

$$[y_C^c, y_C^u, \mathbf{x}_C] \in T_C(\mathbf{k}_C)$$

  where $\mathbf{x}_C = [x_{C1}, x_{C2}, ..., x_{CN_C}]$ is a $1 \times N_C$ vector of variable inputs , $\mathbf{k}_C = [k_{C1}, k_{C2}, ..., k_{CM_C}]$ is a $1 \times M_C$ vector of inputs that are fixed in the short run and where at least some of those inputs are exogenous to the firm and determined by Canadian government policy or programs (e.g. public infrastructure). $T_C(\mathbf{k}_C)$ defines the production technology – i.e. the feasible set of $[y_C^c, y_C^u, \mathbf{x}_C]$ combinations, given $\mathbf{k}_C$ – which respects the conditions necessary for duality

- the U.S. firm similarly produces y_U^c, y_U^u such that

$$[y_U^c, y_U^u, \mathbf{x}_U] \in T_U(\mathbf{k}_U)$$

  where $\mathbf{x}_U$ $[x_{U1}, x_{U2}, ..., x_{UN_U}]$is a $1 \times N_U$ vector of variable inputs, $\mathbf{k}_U = [k_{U1}, k_{U2}, ..., x_{UM_U}]$ is a $1 \times M_U$ vector of inputs that are fixed in the short run – as with the Canadian firm, this will include some inputs that are exogenous to the firm and that are determined by U.S. government policy or programs – and $T_U(\mathbf{k}_U)$ defines the production technology, i.e. the feasible set of $\left[y_U^C, y_U^U, \mathbf{x}_U\right]$ combinations, given $\mathbf{k}_U$ and which also respects the conditions necessary for duality

- Fixed inputs for both firms do not have an effect on interfirm rivalry, i.e. fixed inputs and changes in them are not used in a strategic manner

- Both the Canadian and the U.S. firm take the response of the competition into account when making production/sales decisions – for example, if the Canadian firm plans an increase in sales in the Canadian market, it takes into account the change in sales that will be made by the U.S. firm in the Canadian market. The same considerations apply to the Canadian firm in the U.S. market and the U.S. firm in both the Canadian and U.S. markets. This rivalry can be represented by best response, or reaction, functions. The Canadian firm, which has incomplete information about the U.S. firm's technology, must make an assumption about the U.S. firm's best response to a change in the level of sales of Canadian-produced brands. In particular, the Canadian firm conjectures that the U.S. firm's best response can be represented by the functions $R^c_{U,C}$ and $R^u_{U,C}$,with

  $$y^c_U = R^c_{U,C}(y^c_C) \text{ and } y^u_U = R^u_{U,C}(y^u_C)$$

  The U.S. firm has similar conjectures about the Canadian firm's best responses, i.e. that they can be represented by the functions $R^c_{C,U}$ and $R^u_{C,U}$,with

  $$y^c_C = R^c_{C,U}(y^c_U) \text{ and } y^u_C = R^u_{C,U}(y^u_u)$$

- Both firms are assumed to set their sales levels simultaneously, i.e. there is no leader or follower

- Both firms operate at non-negative profit. This rules out strategic behaviour where a firm deliberately incurs a loss; it also rules out random events that lead to losses

- Inventories are not held – an increase in the amount of a brand available for sale in either market comes from shipments of the commodity produced within the period under consideration – this means that the terms 'sales' and 'production' can be used interchangeably.

3 Firm-level Optimization

3.1 The Canadian Firm

Optimization is based on a two-step procedure, which contrasts with a one-step profit maximization procedure. The two-step approach is characterized in Jehle and Reny(1998, p. 237) as being, in the first step, a calculation made by the firm to determine the least cost of any possible level of production of each brand. In the second step, the firm maximizes the difference between the revenue from any given mix of brands and the cost of producing them. This approach is common in the treatment of oligopoly in the industrial organization literature – see, for example, Waterson(1984, p. 18).

The approach relies on the existence of a multiproduct cost function dual to the technology set $T_C(\mathbf{k}_C)$ that is defined as follows:

$$C_C(\mathbf{w}_C, y_C^c, y_C^u, \mathbf{k}_C) \equiv \min_{\mathbf{x}_C} \left\{ \mathbf{w}_C \mathbf{x}_C^{\mathrm{T}} | \, [y_C^c, y_C^u, \mathbf{x}_C] \in T_C(\mathbf{k}_C); y_C^c > 0; y_C^u > 0 \right\} \tag{1}$$

where $\mathbf{w}_C$ is a $1 \times N_C$ vector of input prices. The problem on the right-hand side is to choose inputs $\mathbf{x}_C$ to minimize the variable cost of producing a given level and mix of production for brands y_C^c and y_C^u , given a particular level of fixed inputs $\mathbf{k}_C$. The set $T_C(\mathbf{k}_C)$ is restricted to ensure that, in the second stage, all solutions are interior solutions. Roughly speaking, this means that boundary points of $T_C(\mathbf{k}_C)$ are excluded.

Given the cost function (1), the associated profit maximization problem, which determines optimal levels of y_C^c and y_C^u, is

$$\max_{y_C^c, y_C^u} \{ p_C^c y_C^c + p_C^u y_C^u - C_C(\mathbf{w}_C, y_C^c, y_C^u, \mathbf{k}_C) \} \tag{2}$$

where, as before, y_C^c and y_C^u are nonzero. When prices p_C^c and p_C^u are replaced by the respective inverse compensated demand functions and the Canadian firm's conjectures about the best response functions replace U.S. firm sales levels, the objective function becomes:

$$\begin{aligned} \max_{y_C^c, y_C^u} \{ & \kappa_C^c(y_C^c, R_{U,C}^c(y_C^c), z_C) y_C^c + \kappa_C^u(y_C^u, R_{U,C}^u(y_C^u), z_U) y_C^u \\ & - C_C(\mathbf{w}_C, y_C^c, y_C^u, \mathbf{k}_C) \} , \end{aligned} \tag{3}$$

with two first-order conditions. The first of these is:

$$p_C^c + \left[\partial \kappa_C^c(\cdot)/\partial y_C^c + (\partial \kappa_C^c(\cdot)/\partial y_U^c)\left(\partial R_{U,C}^c(y_C^c)/\partial y_C^c\right)\right] y_C^c \qquad \text{(4a)}$$
$$-\partial C_C(\mathbf{w}_C, y_C^c, y_C^u, \mathbf{k}_C)/\partial y_C^c = 0$$

where $\kappa_C^c(\cdot)$ denotes $\kappa_C^c(y_C^c, R_{U,C}^c(y_C^c), z_C)$, and p_C^c enters on the left-hand side because it is value of the inverse demand function evaluated at the initial level y_C^c and exogenous level z_C. The term in square brackets on the left-hand side is the result of applying the chain rule - first is the direct effect of marginal change on price p_C^c, while the second is the indirect effect on p_C^c that takes into account the conjectured best response of the U.S. firm to an increase in sales by the Canadian firm in the Canadian market.

The second first-order condition, which can be interpreted in a similar manner, is

$$p_C^u + \left[\partial \kappa_C^u(\cdot)/\partial y_C^u + (\partial \kappa_C^u(\cdot)/\partial y_U^u)\left(\partial R_{U,C}^u(y_C^u)/\partial y_C^u\right)\right] y_C^u \qquad \text{(4b)}$$
$$-\partial C_C(\mathbf{w}_C, y_C^c, y_C^u, \mathbf{k}_C)/\partial y_C^u = 0\ .$$

As it stands, the optimal levels of production/sales y_C^{c*} and y_C^{u*} can only be derived implicitly from (4a) and (4b). Explicit solutions are needed for the competitiveness analysis that follows, so it is necessary to have more specific forms for the inverse compensated demand functions, the best response functions and the cost function. One way to achieve this (no doubt, there are other possibilities) is to make the functions κ_C^c, κ_C^u $R_{U,C}^c$, $R_{U,C}^u$ linear in sales and to assume a more specific form for C_C so that the production variables can be isolated.

Linearity of κ_C^c and κ_C^u means that:

$$p_C^c = \kappa_C^c(y_C^c, y_C^u, z_C) \equiv \alpha_C^c + \xi_{C,C}^c y_C^c + \xi_{C,U}^c y_U^c + \gamma_C^c z_C \qquad \text{(5a)}$$

and

$$p_C^u = \kappa_C^u(y_C^u, y_U^u, z_U) \equiv \alpha_C^u + \xi_{C,C}^u y_C^u + \xi_{C,U}^u y_U^u + \gamma_U^c z_U \qquad \text{(5b)}$$

where $\alpha_C^c, \xi_{C,C}^c, \xi_{C,U}^c, \gamma_C^c, \alpha_C^u, \xi_{C,C}^u, \xi_{C,U}^u$ and γ_U^c are coefficients. These linear forms mean that the partial derivatives in (4a) that relate to demand reduce to

$$\partial\kappa_C^c(\cdot)/\partial y_C^c = \xi_{C,C}^c \text{ and } \partial\kappa_C^c(\cdot)/\partial y_C^u = \xi_{C,U}^c \tag{5c}$$

and those in (4b) reduce to

$$\partial\kappa_C^u(\cdot)/\partial y_C^u = \xi_{C,C}^u \text{ and } \partial\kappa_C^u(\cdot)/\partial y_U^u = \xi_{C,U}^u. \tag{5d}$$

The linear forms for $R_{U,C}^c$ and $R_{U,C}^u$ are

$$y_U^c = R_{U,C}^c(y_C^c) = \beta_U^c + \psi_{U,C}^c y_C^c \tag{6a}$$

and

$$y_U^u = R_{U,C}^u(y_C^u) = \beta_U^u + \psi_{U,C}^u y_C^u \tag{6b}$$

where $\beta_U^c, \beta_U^u, \psi_{U,C}^c$ and $\psi_{U,C}^u$ are coefficients. These linear forms mean that the derivatives in (4a) and (4b) that relate to the conjectured best responses reduce to

$$\partial R_{U,C}^c(y_C^c)/\partial y_C^c = \psi_{U,C}^c\ . \tag{6c}$$

and

$$\partial R_{U,C}^u(y_C^u)/\partial y_C^u = \psi_{U,C}^u. \tag{6d}$$

To obtain a more specific form for C_C, assume that the Canadian firm produces its two brands using a nonjoint technology, where the definition of non-jointness is that given in Hall(1973, p.884).[3] This means that

$$C_C(\mathbf{w}_C, y_C^c, y_C^u, \mathbf{k}_C) \equiv C_C^c(\mathbf{w}_C, \mathbf{k}_C)y_C^c + C_C^u(\mathbf{w}_C, \mathbf{k}_C)y_C^u \tag{7}$$

where $C_C^c(\mathbf{w}_C, \mathbf{k}_C)$ is the marginal cost of producing the brand sold in Canada and $C_C^u(\mathbf{w}_C, \mathbf{k}_C)$ is the marginal cost of producing the brand sold in the United States. Note that this form does not necessarily mean that there are

[3]The nonjoint technology means that the underlying technology specified earlier as $[y_C^c, y_C^u, \mathbf{x}_C] \in T_C(\mathbf{k}_C)$ is replaced by a more specific form. For the non-joint technology assumed here there are individual production functions f_C^c and f_C^u such that

$$y_C^c = f_C^c(\mathbf{x}_C^c, \mathbf{k}_C) \text{ and } y_C^u = f_C^u(\mathbf{x}_C^u, \mathbf{k}_C)$$

– see Hall(1973, p.884). Notice that there are no economies (or diseconomies) of scope with this type of technology.

distinct and physically separate processes used to produce the two brands. This is reflected in part by the vector $\mathbf{k}_C$; no element of this vector is specifically allocated to one brand or the other.

For each brand, marginal cost is independent of the level of production (and of the mix of brands produced – see Hall(1973, p.885):[4]

$$\partial C_C(\mathbf{w}_C, y_C^c, y_C^u, \mathbf{k}_C)/\partial y_C^c = C_C^c(\mathbf{w}_C, \mathbf{k}_C) \tag{8a}$$

and

$$\partial C_C(\mathbf{w}_C, y_C^c, y_C^u, \mathbf{k}_C)/\partial y_C^u = C_C^u(\mathbf{w}_C, \mathbf{k}_C). \tag{8b}$$

With these simplifying assumptions, it is now possible to express the optima y_C^{c*} and y_C^{u*} in terms of the other variables and coefficients. Substitution of (6c), (8a) and elements of (5c), into (4a) and rearrangement to isolate y_C^c gives:

$$y_C^{c*} = \frac{C_C^c(\mathbf{w}_C, \mathbf{k}_C) - p_C^c}{\Phi_C^c} \tag{9a}$$

where $\Phi_C^c = \xi_{C,C}^c + \xi_{C,U}^c \psi_{U,C}^c$ and where, for this to be a 'well-behaved' supply equation, $\partial y_C^{c*}/\partial p_C^c > 0$, meaning that $\Phi_C^c < 0$ must hold, so that $\xi_{C,U}^c \psi_{U,C}^c < \left|\xi_{C,C}^c\right|$, since by the concavity of the inverse compensated demand function, $\xi_{C,C}^c < 0$. Since, by homogeneity, $\xi_{C,U}^c > 0$, a sufficient (and unnecessarily restrictive) condition for $\Phi_C^c < 0$ to hold is that $\psi_{U,C}^c < 0$.

Similarly, substitution of (6d), (8b) and elements of (5d) into (4b) and rearrangement to isolate y_C^u gives

[4] If marginal are equal for the two brands, then

$$\begin{aligned}\partial C_C(\mathbf{w}_C, y_C^c, y_C^u, \mathbf{k}_C)/\partial y_C^c &= \partial C_C(\mathbf{w}_C, y_C^c, y_C^u, \mathbf{k}_C)/\partial y_C^u \\ &= C_C(\mathbf{w}_C, \mathbf{k}_C)\ .\end{aligned}$$

If they differ by some fixed proportion,

$$\begin{aligned}\partial C_C(\mathbf{w}_C, y_C^c, y_C^u, \mathbf{k}_C)/\partial y_C^c &= C_C(\mathbf{w}_C, \mathbf{k}_C) \text{ and} \\ \partial C_C(\mathbf{w}_C, y_C^c, y_C^u, \mathbf{k}_C)/\partial y_C^u &= C_C(\mathbf{w}_C, \mathbf{k}_C)(1+\alpha)\end{aligned}$$

where α is some real number. For example, if $\alpha = 0.05$, the marginal cost of the the brand sold in the U.S. is 5% higher than the marginal cost of the brand sold in Canada.

$$y_C^{u*} = \frac{C_C^u(\mathbf{w}_C, \mathbf{k}_C) - p_C^u}{\Phi_C^u}. \tag{9b}$$

where $\Phi_C^u = \xi_{C,C}^u + \xi_{C,U}^u \psi_{U,C}^u$ and, following the argument made above, it must be that $\Phi_C^u < 0$, and therefore that $\xi_{C,U}^u \psi_{U,C}^u < \left|\xi_{C,C}^u\right|$.

3.2 The U.S. Firm

For the U.S. firm, the problem is the same as that for the Canadian firm. Nevertheless, for clarity, this is fully specified in a similar manner. In particular, for the U.S. firm, the multiproduct cost function dual to its technology set $T_U(\mathbf{k}_U)$ is:

$$C_U(\mathbf{w}_U, y_U^c, y_U^u, \mathbf{k}_U) \equiv \min_{\mathbf{x}_U} \left\{ \mathbf{w}_U \mathbf{x}_U^{\mathrm{T}} \middle| \, [y_U^c, y_U^u, \mathbf{x}_U] \in T_U(\mathbf{k}_U); y_U^c > 0, y_U^u > 0 \right\} \tag{10}$$

where $\mathbf{w}_U$ is a $1 \times N_U$ vector of input prices. The problem is the same as that of the Canadian firm – choose inputs $\mathbf{x}_U$ to minimize the variable cost of producing a given level and mix of brands $y_U^c > 0$ and $y_U^u > 0$, given a particular level of fixed inputs $\mathbf{k}_U$.

Since it is clear from earlier discussion that the U.S. firm's technology must be nonjoint, assume, as for the Canadian firm, that:

$$C_U(\mathbf{w}_U, y_U^c, y_U^u, \mathbf{k}_U) \equiv C_U^c(\mathbf{w}_U, \mathbf{k}_U) y_U^c + C_U^u(\mathbf{w}_U, \mathbf{k}_U) y_U^u \tag{11}$$

where $C_U^c(\mathbf{w}_U, \mathbf{k}_U)$ is the marginal cost of producing the brand sold in Canada and $C_U^u(\mathbf{w}_U, \mathbf{k}_U)$ is the marginal cost of producing the brand sold in the United States.

The associated profit maximization problem is then

$$\max_{y_U^c, y_U^u} \{ p_U^c y_U^c + p_U^u y_U^u - C_U^c(\mathbf{w}_U, \mathbf{k}_U) y_U^c - C_U^u(\mathbf{w}_U, \mathbf{k}_U) y_U^u \} \tag{12}$$

which can be re-expressed as, using inverse compensated demand functions and the conjectured Canadian firm's best response functions $R_{C,U}^c$ and $R_{C,U}^u$

$$\max_{y_U^c, y_U^u} \left\{ \kappa_U^c(R_{C,U}^c(y_U^c), y_U^c, z_C) y_U^c + \kappa_U^u(R_{C,U}^u(y_U^u), y_U^u, z_U) y_U^u - C_U^c(\mathbf{w}_U, \mathbf{k}_U) y_U^c \right.$$
$$\left. - C_U^u(\mathbf{w}_U, \mathbf{k}_U) y_U^u \right\} . \tag{13}$$

The first of the two first-order conditions is:

$$p_U^c + \left[\partial\kappa_U^c(\cdot)/\partial y_U^c + (\partial\kappa_U^c(\cdot)/\partial y_C^c) \left(\partial R_{C,U}^c(y_U^c)/\partial y_U^c \right) \right] y_U^c$$
$$- C_U^c(\mathbf{w}_U, \mathbf{k}_U) = 0 \tag{14a}$$

where the $(\cdot)$ notation is used in the same manner as for the Canadian firm derivations

The second first-order condition, which can be interpreted in a similar manner, is

$$p_U^u + \left[\partial\kappa_U^u(\cdot)/\partial y_U^u + (\partial\kappa_C^u(\cdot)/\partial y_C^u) \left(\partial R_{C,U}^u(y_U^u)/\partial y_U^u \right) \right] y_U^u$$
$$- C_U^u(\mathbf{w}_U, \mathbf{k}_U) = 0 . \tag{14b}$$

As for the Canadian firm, the functions κ_U^c, κ_U^u $R_{C,U}^c$, $R_{C,U}^u$ are assumed to be linear in sales. In particular, the linear inverse compensated demand functions facing the U.S. firm are:

$$p_U^c = \kappa_U^c(y_C^c, y_U^c, z_C) \equiv \alpha_U^c + \xi_{U,C}^c y_C^c + \xi_{U,U}^c y_U^c + \gamma_U^c z_C \tag{15a}$$

$$p_U^u = \kappa_U^u(y_C^u, y_U^u, z_U) \equiv \alpha_U^u + \xi_{U,C}^u y_C^u + \xi_{U,U}^u y_U^u + \gamma_U^u z_U \tag{15b}$$

where $\alpha_U^c, \xi_{U,C}^c, \xi_{U,U}^c, \gamma_U^c, \alpha_U^u, \xi_{U,C}^u, \xi_{U,U}^u$, and γ_U^u are coefficients. This means that the partial derivatives with respect to demand that appear in (14a) are simply

$$\partial\kappa_U^c(\cdot)/\partial y_C^c = \xi_{U,C}^c \text{ and } \partial\kappa_U^c(\cdot)/\partial y_U^c = \xi_{U,U}^c \tag{15c}$$

and those in (14b) are

$$\partial \kappa_U^u(\cdot)/\partial y_C^u = \xi_{U,C}^u \text{ and } \partial \kappa_U^u(\cdot)/\partial y_U^u = \xi_{U,U}^u \tag{15d}$$

The linear forms for $R_{C,U}^c$ and $R_{C,U}^u$ are similarly

$$y_C^c = R_{C,U}^c(y_U^c) = \beta_C^c + \psi_{C,U}^c y_U^c \tag{16a}$$

and

$$y_C^u = R_{C,U}^u(y_U^u) = \beta_C^u + \psi_{C,U}^u y_U^u \tag{16b}$$

where $\beta_C^c, \beta_C^u, \psi_{C,U}^c$ and $\psi_{C,U}^u$ are coefficients, so the best response elements of (4a) and (4b) reduce to

$$\partial R_{C,U}^c(y_U^c)/\partial y_U^c = \psi_{C,U}^c \tag{16c}$$

and

$$\partial R_{C,U}^u(y_U^u)/\partial y_U^u = \psi_{C,U}^u \tag{16d}$$

Substitution of (16c) and elements of (15c) into (14a) and rearrangement gives the optimal level of production/sales by the U.S. firm in the Canadian market

$$y_U^{c*} = \frac{C_U^c(\mathbf{w}_U, \mathbf{k}_U) - p_U^c}{\Phi_U^c} \tag{17a}$$

where $\Phi_U^c = \xi_{U,U}^c + \xi_{U,C}^c \psi_{C,U}^c$ and where, for $\partial y_U^{c*}/\partial p_U^c > 0$ to hold, $\Phi_U^c < 0$, so $\xi_{U,C}^c \psi_{C,U}^c < \left|\xi_{U,U}^c\right|$, given that $\xi_{U,U}^c < 0$ by concavity.

Similarly, substitution of (16d) and elements of (15d) and (14b) rearrangement gives the optimal level of sales by the U.S. firm in the U.S. market

$$y_U^{u*} = \frac{C_U^u(\mathbf{w}_U, \mathbf{k}_U) - p_U^u}{\Phi_U^u}. \tag{17b}$$

where $\Phi_U^u = \xi_{U,U}^u + \xi_{U,C}^u \psi_{C,U}^u$, $\Phi_U^u < 0$ so $\xi_{U,C}^u \psi_{C,U}^u < \left|\xi_{U,U}^u\right|$.

3.3 Equilibrium

Note that the equilibrium price for each brand in each market is determined through the inverse compensated demand functions (5a),(5b),(15a) and (15b) evaluated at the optimum levels of production/sales $y_C^{c*}, y_U^{c*}, y_U^{c*}$ and y_U^{u*} respectively. The model is thus comprised of two simultaneous equation systems. The first system, which applies to the Canadian market, is made up of equations (5a),(9a),(15a) and (17a). The second system, which applies to the U.S. market, is comprised of equations (5b),(9b),(15b) and (17b). The exogenous variables that are common to both systems are the input price vectors $\mathbf{w}_C, \mathbf{w}_U$, the fixed input vectors $\mathbf{k}_C$ and $\mathbf{k}_U$ and the fixed levels of subutility z_C and z_U.

Note that equilibrium requires that the Canadian firm's conjectures about the U.S. firm's best responses and the U.S. firm's conjectures about the Canadian firm's best responses are, in fact, borne out. In particular, it must be that:
$y_C^{c*} = R_{C,U}^c(y_U^{c*}), y_C^{u*} = R_{C,U}^u(y_U^{u*}), y_U^{c*} = R_{U,C}^c(y_C^{c*})$ and $y_U^{u*} = R_{U,C}^u(y_C^{u*})$.

The solutions to the second system (U.S. market) is now derived – solutions for the first system (Canadian market) are identical, differing only in notation. The second system can be expressed in matrix form, with exogenous components on the right-hand side, i.e.

$$\underbrace{\begin{bmatrix} 1 & 0 & 1/\Phi_C^u & 0 \\ 0 & 1 & 0 & 1/\Phi_U^u \\ -\xi_{C,C}^u & -\xi_{C,U}^u & 1 & 0 \\ -\xi_{U,C}^u & -\xi_{U,U}^u & 0 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} y_C^{u*} \\ y_U^{u*} \\ p_C^u \\ p_U^u \end{bmatrix}}_{B} = \underbrace{\begin{bmatrix} C_C^u(\mathbf{w}_C, \mathbf{k}_C)/\Phi_C^u \\ C_U^u(\mathbf{w}_U, \mathbf{k}_U)/\Phi_U^u \\ \alpha_C^u + \gamma_C^u z_U \\ \alpha_U^u + \gamma_U^u z_U \end{bmatrix}}_{C} \tag{18}$$

where A is the 4×4 matrix of coefficients, B is the 4×1 vector of endogenous variables (sales by the Canadian firm of its U.S. brand in the U.S. market, sales by the U.S. firm of its U.S. brand in the U.S. market, and prices of each brand respectively) and C is the 4×1 vector of exogenous components. The vector B can be solved for by calculating A^{-1} and deriving $A^{-1}C$.The derived expression A^{-1} is given in the Appendix. The expression for y_C^{u*} resulting from these derivations is:

$$y_C^{u*} = (1/|A|) \left\{ \left[1 + \xi_{U,U}^u/\Phi_U^u\right] \left[C_C^u(\mathbf{w}_C, \mathbf{k}_C)/\Phi_C^u\right] \right.$$
$$- \left[\xi_{C,U}^u/\Phi_C^u\right] \left[C_U^u(\mathbf{w}_U, \mathbf{k}_U)/\Phi_U^u\right]$$
$$- \left[1/\Phi_C^u + \xi_{U,U}^u/\left(\Phi_C^u\Phi_U^u\right)\right] \left[\alpha_C^u + \gamma_C^u z_U\right]$$
$$\left. + \left[\xi_{C,U}^u/\left(\Phi_C^u\Phi_U^u\right)\right] \left[\alpha_U^u + \gamma_U^u z_U\right] \right\} \tag{19a}$$

where

$$|A| = 1 + \xi_{U,U}^u/\Phi_U^u + \xi_{C,C}^u/\Phi_C^u + \xi_{C,C}^u\xi_{U,U}^u/(\Phi_U^u\Phi_C^u) - \xi_{C,U}^u\xi_{U,C}^u/(\Phi_U^u\Phi_C^u)$$

Note that, providing the conditions established earlier, i.e. that

$$\xi_{C,C}^u < 0, \xi_{U,U}^u < 0, \xi_{C,U}^u > 0, \xi_{U,C}^u > 0, \Phi_C^u < 0, \Phi_U^u < 0,$$

$|A|$ will be positive and non-zero if

$$1 + \xi_{U,U}^u/\Phi_U^u + \xi_{C,C}^u/\Phi_C^u + \xi_{C,C}^u\xi_{U,U}^u/(\Phi_U^u\Phi_C^u) > \xi_{C,U}^u\xi_{U,C}^u/(\Phi_U^u\Phi_C^u)\ .$$

It will be assumed that this inequality applies (the necessary and/or sufficient conditions under which this might hold could be worked out).

Notice that y_C^{u*} depends not only on all of the variables/coefficients related to the Canadian firm and U.S. consumer, but also on all of the variables/coefficients related to the U.S. firm. It is therefore important to take this interdependence into account when looking at the impact of any policy or program on the Canadian firm – there will be spillover to the U.S. firm (see below) and changes that come about from its production/sales response.

The expression for y_U^{u*} in terms only of exogenous variables is

$$y_U^{u*} = (1/|A|) \left\{ \left[-\xi_{U,C}^u/\Phi_U^u\right] \left[C_C^u(\mathbf{w}_C, \mathbf{k}_C)/\Phi_C^u\right] \right.$$
$$+ \left[1 + \xi_{C,C}^u/\Phi_C^u\right] \left[C_U^u(\mathbf{w}_U, \mathbf{k}_U)/\Phi_U^u\right]$$
$$+ \left[\xi_{U,C}^u/\left(\Phi_C^u\Phi_U^u\right)\right] \left[\alpha_C^u + \gamma_C^u z_U\right]$$
$$\left. - \left[1/\Phi_U^u - \xi_{C,C}^u/\left(\Phi_C^u\Phi_U^u\right)\right] \left[\alpha_U^u + \gamma_U^u z_U\right] \right\}. \tag{19b}$$

4 Measuring an Improvement in Competitiveness

It is now possible to use the solutions derived above to examine the conditions under which a policy change can generate an improvement in the competitiveness of the Canadian firm. To begin, it is useful to reiterate the definition of competitiveness stated in the Introduction, i.e. that

> For a firm, an improvement in competitiveness will be reflected by consistent sales growth in any relevant market at a rate that is at least as high as the sales growth of other firms (the competition) in that market.

This means that it is necessary to think of competitiveness in terms of differences in optimal levels of sales between the two firms that arise from changes in one or more exogenous variable for one or both firms. The model, however, is a static one and cannot accommodate growth in a direct way. It is instead necessary to think of changes in variables that are exogenous to both firms, and to look at how the optimal level of production/sales changes in each case. These changes can be viewed as one-time increases or decreases in sales where new levels are maintained until there is another change in one or more exogenous variable.

Consider now an increase in the quantity of one of the fixed inputs available to the Canadian firm. Suppose that, of the M_C fixed inputs, one is the stock of public road infrastructure – let this be the element k_{C5} in the vector $\mathbf{k}_C$. Now suppose that the stock increases, for example because the Canadian government has funded the construction or improvement of a road used by the Canadian firm. This will have a direct cost-reducing effect for the Canadian firm since allows the firm to decrease the level of one or more inputs (such as fuel) even though it increases production/sales.

To look at the impact of this investment on the competitiveness of the Canadian firm, first note that the total differential of the marginal cost function $C_C^u(\mathbf{w}_C, \mathbf{k}_C) is$

$$\mathrm{d}C_C^u(\mathbf{w}_C, \mathbf{k}_C) \equiv \sum\nolimits_{i=1}^{N_C} \lambda_{C\,i} \mathrm{d}w_{C\,i} + \sum\nolimits_{j=1}^{M_C} \mu_{Cj}^u \mathrm{d}k_{Cj} \tag{20}$$

where $\lambda_{C\,i} = \partial C^u_C(\cdot)/\partial w_{C\,i}$ and $\mu^u_{Cj} = \partial C^u_C(\cdot)/\partial k_{C\,j} < 0 \forall j$; μ^u_{Cj} is the shadow value of a fixed input $k_{C\,j}$, i.e. the reduction in the marginal cost of the U.S. brand produced by the Canadian firm that arises from a marginal increase in that fixed input.[5] Note that there will be an effect $\mu^c_{Cj} = \partial C^c_C(\cdot)/\partial k_{C\,j}$ for the Canadian brand as well, but that it will not necessarily be true that $\mu^c_{Cj} = \mu^u_{Cj}$ either due to $k_{C\,j}$ being exogenous to the firm, because it is not possible to allocate a fixed input across the production of brands in such a way that these shadow values are equal.

If all other exogenous variables are held constant (i.e. all Canadian and U.S. input prices, all fixed inputs for the U.S. firm, the subutility levels for the Canadian and U.S. consumers, and all but the fifth fixed input for the Canadian firm),

$$\begin{aligned} \mathrm{d}y^{u*}_C &= (1/|A|)\left[1 + \xi^u_{U,U}/\Phi^u_U\right]\left[\mathrm{d}C^u_C(\mathbf{w}_C, \mathbf{k}_C)/\Phi^u_C\right] \\ &\equiv (1/|A|)\left[1 + \xi^u_{U,U}/\Phi^u_U\right](\mu^u_{C5}/\Phi^u_C)\mathrm{d}k_{C5} \end{aligned} \tag{21a}$$

and

$$\begin{aligned} \mathrm{d}y^{u*}_U &= (1/|A|)\left[-\xi^u_{U,C}/\Phi^u_U\right]\left[\mathrm{d}C^u_C(\mathbf{w}_C, \mathbf{k}_C)/\Phi^u_C\right] \\ &\equiv (1/|A|)\left[-\xi^u_{U,C}/\Phi^u_U\right](\mu^u_{C5}/\Phi^u_C)\,\mathrm{d}k_{C5}. \end{aligned} \tag{21b}$$

Note that the new equilibrium level of sales for the two firms ($y^{u*}_C + \mathrm{d}y^{u*}_C$ and $y^{u*}_U + \mathrm{d}y^{u*}_U$) is reached with each firm using its conjecture about the other's best response function in order to determine the new optimal level of sales. Also note that both $\mathrm{d}y^{u*}_C$ and $\mathrm{d}y^{u*}_U$ are positive, so the public investment in Canadian infrastructure leads to an expansion in U.S. production and sales by the U.S. firm, even though that firm does not actually use Canadian

[5] Note that $\lambda_{C\,i}$ does not give the i^{th} input demand, since Shephard's lemma only applies to the total cost function $C_C(\mathbf{w}_C, y^c_C, y^u_C, \mathbf{k}_C)$, i.e.

$$\begin{aligned} x_{C\,i}(\mathbf{w}_C, y^c_C, y^u_C, \mathbf{k}_C) &= \partial C_C(\mathbf{w}_C, y^c_C, y^u_C, \mathbf{k}_C)/\partial w_{C\,i} \\ &\equiv (\partial C^c_C(\mathbf{w}_C, \mathbf{k}_C)/\partial w_{C\,i})\, y^c_C + (\partial C^u_C(\mathbf{w}_C, \mathbf{k}_C)/\partial w_{C\,i})\, y^u_C \end{aligned}$$

This illustrates the fact that the actual input demands per brand cannot be determined, only the total input demand for both brands, even though the technology is non-joint.

roads.[6] This could be interpreted of a positive spillover that the U.S. firm receives as a result of the investment made by the Canadian government.

Then the 'bottom line' question is, to what extent will this new investment in infrastructure lead to an improvement in the competitiveness of the Canadian firm? This will be reflected by an increase in sales for the Canadian firm that is at least high as that of the U.S. firm. To determine the conditions under which this might occur, subtract (21b) from (21a) to get:

$$\begin{aligned}\Delta_C^u = \mathrm{d}y_C^{u*} - \mathrm{d}y_U^{u*} &\equiv (1/|A|)\left[1 + \xi_{U,U}^u/\Phi_U^u + \xi_{U,C}^u/\Phi_U^u\right](\mu_{C5}/\Phi_C^u)\,\mu_{C5}\mathrm{d}k_{C5} \\ &\equiv (1/|A|)\left[\Phi_U^u + \xi_{U,U}^u + \xi_{U,C}^u\right]\mu_{C5}\mathrm{d}k_{C5}/(\Phi_U^u\Phi_C^u)\ ,\end{aligned} \tag{22}$$

where Δ_C^u denotes the change in competitiveness of the Canadian firm relative to the U.S. firm in the U.S. market. Since $(1/|A|)\mu_{C5}\mathrm{d}k_{C5}/(\Phi_U^u/\Phi_C^u) < 0$, $\Delta_C^u > 0$ only if

$$\Phi_U^u + \xi_{U,U}^u + \xi_{U,C}^u < 0. \tag{23a}$$

To determine the conditions under which this inequality will hold, substitute $\xi_{U,U}^u + \xi_{U,C}^u\psi_{C,U}^u$ for Φ_U^u and use the homogeneity property of the inverse compensated demand function to replace $\xi_{U,U}^u$ with $-(y_C^u/y_U^u)\xi_{U,C}^u$. Then divide through the resulting inequality by $\xi_{U,C}^u$ (which is always positive). These changes give the following result:

$$\Delta_C^u > 0 \text{ only if } \psi_{C,U}^u < 2y_C^u/y_U^u - 1. \tag{23b}$$

Whether or not (23b) holds depends upon the relative magnitudes of y_C^u/y_U^u and $\psi_{C,U}^u$, i.e. the ratio of Canadian to U.S. sales and the best response of the Canadian firm as conjectured by the U.S. firm. This latter parameter reflects the rivalrous aspect of the model – so long as the U.S. firm conjectures a best response that falls below $2y_C^u/y_U^u - 1$ and sets its level of sales accordingly, there will be an improvement in competitiveness for the Canadian firm.

[6] One extension would be to allow certain fixed inputs, such as infrastructure, to appear in both the Canadian and U.S. firms's cost function. This would be appropriate if, when the U.S. ships its brand for the Canadian market into Canada, it uses Canadian infrastructure and vice versa. As things stand, the U.S. firm's Canadian sales could be thought of as occuring through a broker that arranges transport of the commodity from the U.S. plant to the Canadian market.

Figure 1 illustrates the set of values of $\psi^u_{C,U}$ that corresponds to the range $y^u_C/y^u_U \in (0,1)$ for which (23b) will hold, i.e. in the range where both firms are making sales in the U.S. market. The shaded area gives the set of $(y^u_C/y^u_U, \psi^u_{C,U})$ pairs where improvements in competitiveness can occur. The line is the set of threshold combinations, i.e. the maximum value of $\psi^u_{C,U}$ for each level of y^u_C/y^u_U where $\Delta^u_C > 0$. It is important to note that, for pairs $\left(y^u_C/y^u_U, \psi^u_{C,U}\right)$ outside of this set, competitiveness of the Canadian firm will actually deteriorate. Thus, if for example, $y^u_C/y^u_U = 0.4$ (the Canadian firm has sales that are 40% as high as the U.S. firm's) and $\psi^u_{C,U} = -0.1$, the investment in public infrastructure by the Canadian government will have a unintended negative impact on the competitiveness of the Canadian firm. This result is consistent for the most part with Bulow et al., who find in their two-firm/two- market/two-commodity model that a subsidy to one of the firms in the first market will increase the other firm's activity in the second market, hurting the first firm in that market – see Bulow, Geanakopolos and P.D. Klemperer (1983. p. 25).

Suppose that (23b) holds so that there is an improvement in competitiveness for the Canadian firm. Then the magnitude of that increase will depend upon

- the degree to which the U.S. firm's conjecture of the Canadian firm's best response $(\psi^u_{C,U})$ falls below $2y^u_C/y^u_U - 1$. For any ratio $y^u_C/y^u_U \in (0,1)$, this means that if $\psi^u_{C,U}$ is less than the threshold value (this is the boundary line in Figure 1) the improvement in competitiveness will be higher than if it were equal to the threshold value. In other words, the Canadian firm's improvement in competitiveness will be larger to the extent that the U.S. firm's conjecture about the Canadian firms's best response is higher. This applies both in the range where threshold value of the conjectured best response is negative – i.e. in the range $y^u_C/y^u_U \in (0, 0.5)$ – and in the range of values where this is nonnegative i.e. the range $y^u_C/y^u_U \in [0.5, 1)$. [7]

[7] The former case, where the threshold value of $\psi^u_{C,U}$ is negative, appears to be consistent with the notion of strategic substitute, i.e. in this range, the best response by the Canadian firm to an increase in y^u_U (increase in sales by the U.S. firm in the U.S market) is to decrease its own sales (decrease y^u_C) so that the U.S. brand is substituted for the Canadian brand.

The latter case, where the threshold value of $\psi^u_{C,U}$ is nonnegative, appears to be con-

- the size of $\mu_{C5}/(\Phi_U^u \Phi_C^u) < 0$, the weighted shadow value related to the marginal cost of of the brand sold on the U.S. market by the Canadian firm. Note that the denominator is a composite comprised of: (i) the coefficients $\xi_{C,C}^u, \xi_{C,U}^u$.from the U.S. consumer's inverse compensated demand function for the Canadian brand; (ii) the coefficients $\xi_{U,U}^u, \xi_{U,C}^u$.from the U.S. consumer's inverse compensated demand function for the U.S. brand; (iii) the Canadian firm's conjecture about the U.S. firm's best response, $\psi_{U,C}^u$; and (iv) the U.S. firm's conjecture about the Canadian firm's best response, $\psi_{C,U}^u$. The smaller the term $\Phi_U^u \Phi_C^u$, the larger the effect of the infrastructure investment on the Canadian firm's competitiveness.

- the size of the determinant $|A|$ – the smaller this is, the larger will be Δ_C^u; since $|A|$ is comprised of all of the relevant coefficients from the inverse compensated demand functions and the conjectured best responses, its size will depend upon the relative magnitude of all of these coeffients.

- the size of $\mathrm{d}k_{C5}$,the infrastructure investment made by the Canadian government.

Taken together, these results show that, while an investment in Canadian infrastructure will lead to an increase in sales of the brand the Canadian firm produces for the U.S. market, this need not translate into an improvement in competitiveness for the Canadian firm. If there is an improvement in competitiveness, moreover, the size of this will depend on U.S. consumer response and the conjectures made by both firms about their best responses.

5 Conclusion

The model developed here provides several insights regarding the nature of competitiveness and the potential role for government policy in improving it. The approach is simple: a two-market duopoly with a Canadian firm selling two brands – one in the U.S. market and one in the Canadian market; the U.S. firm also sells two brands, one in the U.S. market and one brand in

sistent with the notion of strategic complements where an increase in y_U^u.is met by an increase in y_C^u – see Bulow, Geanakopolos and P.D. Klemperer (1983. pp. 2,8).

the Canadian market. Consumers determine market prices by their demand choices, which are modelled using compensated inverse demand functions. The firm's production costs are modelled using an assumed non-joint technology, which nevertheless retains the link that both firms have with the two markets. By incorporating consumer demand and inter-firm rivalry, it is possible to develop a large variety of testable hypotheses regarding the conditions under which competitiveness will improve.

It is shown a cost-reducing investment in public infrastructure made by the Canadian government may lead to an improvement in the competitiveness of the Canadian firm, but that this is not certain – policy changes that on the surface look like sure bets can actually lead to a deterioration in competitiveness. The effect of the investment on the competitiveness of the Canadian firm in the U.S. market is examined. An improvement in competitiveness depends on two factors: relative production/sales prior to the policy change and the U.S. firm's conjecture about the best response of the Canadian firm to an increase in its output. Where an improvement in competitiveness does happen, its size depends on the relative magnitude of the coefficients in the U.S. consumer inverse demand functions and the conjectured best response coefficients of both firms.

Only the competitiveness of the Canadian firm in the U.S. market is examined. This follows from the tendency to think in terms of the competitiveness of Canadian firms on world (export) markets rather than their competitiveness relative to foreign firms making sales on the Canadian market. Both markets are relevant and the model can capture both effects simultaneously. It is conceivable that competitiveness increases in one market but not in the other and the conditions under which this might occur have not been explored.

It is also possible to measure the effect of policy changes not only on competitiveness but also on profits. It is likely that, under certain conditions, there could be an expansion in sales/production in both markets but a decline in overall profits, so examination of these conditions and the likelihood of them would be useful.

The role of costs in competitiveness is important. Within the model, changes in costs come about through changes in input prices or in levels of fixed inputs. While the latter aspect was investigated, the former was not.

It would be useful to look at the impact of relative input price changes in some detail.

It may be possible to estimate the model using firm-level or even industry-level data, or at least generate some values for the various coefficients. Even if the model cannot be estimated, the results obtained here suggest that it is still possible to develop a better understanding of the areas where policies can have the most effect. The model could also be useful in identifying the types of data and other information that need to be collected to better understand the potential impact of policies on the competitiveness of Canadian firms.

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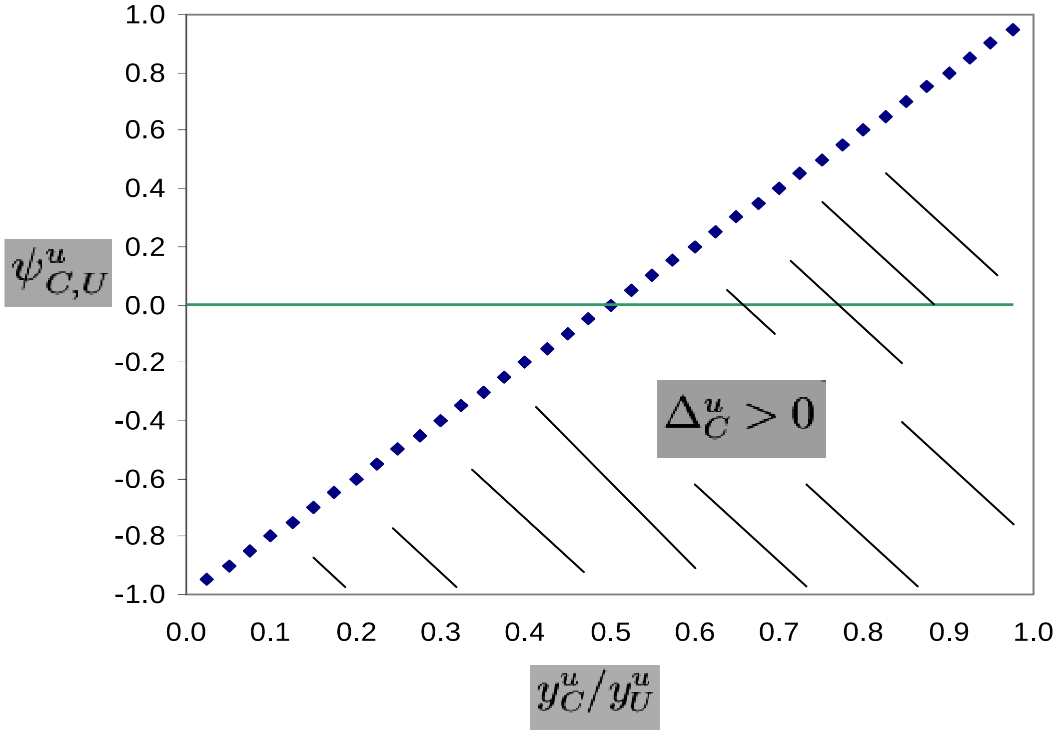
Figure1


1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
$\psi^u_{C,U}$
$\Delta^u_C > 0$
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
y^u_C / y^u_U

Appendix: A inverse

$$A^{-1} = (1/|A|)\times \begin{bmatrix} 1+\xi^u_{U,U}/\Phi^u_U & -\xi^u_{C,U}/\Phi^u_C & \begin{array}{c}-1/\Phi^u_C \\ -\xi^u_{U,U}/\left(\Phi^u_C\Phi^u_U\right)\end{array} & \xi^u_{C,U}/\left(\Phi^u_C\Phi^u_U\right) \\ -\xi^u_{U,C}/\Phi^u_U & 1+\xi^u_{C,C}/\Phi^u_C & \xi^u_{U,C}/\left(\Phi^u_C\Phi^u_U\right) & \begin{array}{c}-1/\Phi^u_U \\ -\xi^u_{C,C}/\left(\Phi^u_C\Phi^u_U\right)\end{array} \\ \begin{array}{c}\xi^u_{C,C}-\xi^u_{C,U}\xi^u_{U,C}/\Phi^u_U \\ +\xi^u_{C,C}\xi^u_{U,U}/\Phi^u_U\end{array} & \xi^u_{C,U} & 1+\xi^u_{U,U}/\Phi^u_U & -\xi^u_{C,U}/\Phi^u_U \\ \xi^u_{U,C} & \begin{array}{c}\xi^u_{U,U}-\xi^u_{C,U}\xi^u_{U,C}/\Phi^u_C \\ +\xi^u_{C,C}\xi^u_{U,U}/\Phi^u_C\end{array} & -\xi^u_{U,C}/\Phi^u_C & 1+\xi^u_{C,C}/\Phi^u_C \end{bmatrix}$$

where

$$|A| = 1+\xi^u_{U,U}/\Phi^u_U+\xi^u_{C,C}/\Phi^u_C+\xi^u_{C,C}\xi^u_{U,U}/(\Phi^u_U\Phi^u_C)-\xi^u_{C,U}\xi^u_{U,C}/(\Phi^u_U\Phi^u_C)$$